

# Comment on chiral symmetry restoration at finite density in large- $N_c$ QCD

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In the article “On chiral symmetry restoration at finite density in large- $N_c$  QCD” by Adhikari, Cohen, Ayyagari and Strother [Phys. Rev. C **83**, 065201 (2011)] the description of dense nuclear matter by means of Skyrmions in hyperspherical unit cells is severely criticized. We point out that this criticism is based on invalid assumptions and therefore unwarranted.

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Motivated by the suggested quarkyonic phase of dense matter at large  $N_c$  [1], Adhikari et al. study in Ref. [2] conditions for chiral symmetry restoration at high baryon density in Skyrme models [3] and in large- $N_c$  QCD. Section IV of their paper is devoted to the “hypersphere approach” [4, 5] which models certain aspects of dense matter (in particular the transition to chiral restoration) by placing Skyrmions in cells with the geometry of a three-dimensional sphere  $S^3$ . Adhikari et al.’s discussion culminates in the claim that the results of the  $S^3$  approach are artifacts of a supposedly arbitrary choice for the cell geometry from which not even qualitative physical insight can be gained. The purpose of the following brief comment is to refute this criticism by identifying a misconception and several incorrect assumptions [6] which underly Ref. [2]’s arguments. (Hence no additional interpretation or justification for the  $S^3$  approach will be required.)

The first of these assumptions is that the hypersphere approach was intended to “approximate a Skyrmion in the crystal”. The second, more drastic one is that the cell shape is insignificant and motivated by practical convenience only: “Of course, this geometry has no significance and was used for ease of computation” (p. 10 of Ref. [2]). In addition, Ref. [2] assumes that the  $S^3$  approach is based on the premise “that the principal effect of putting a Skyrmion into a crystal is to restrict the space over which it can spread” and “that using a hypersphere to restrict the volume of the Skyrmion acts generically like other restrictions on its volume”.

Although the above assumptions were implied in Ref. [2] to be commonly accepted, they are (to the best of our knowledge) nowhere stated in the hypersphere literature. In fact, especially the second and third assumption are in plain contradiction with the unique symmetry properties of the  $S^3$  cell. These two assumptions deny precisely the indispensable role of the  $S^3$  geometry which, due to its “chiral” symmetry group  $SO(4) \simeq SU(2) \times SU(2)$ , made the approach promising in the first place [4–6]. Indeed, already the pioneering papers [4, 5] pointed out that *only* in the  $S^3$  geometry both the chirally-broken and (in an averaged sense) restored phases can be modeled, that a transition between them occurs at a critical energy and baryon density, that the Skyrmion can attain its minimal energy only on  $S^3$ , that parity doubling (including that of the former Goldstone pion triplet) takes place in the

restored phase as expected from complete chiral restoration etc.. (The unique, curvature-generated interactions in  $S^3$  cells [11] were tentatively interpreted as modeling dense-matter-induced chiral forces [5].)

Furthermore, one should not regard hyperspherical cells as faithful models of flat crystal unit-cells, several analogies and rather closely shared results notwithstanding. In fact, the qualitative differences between the two cell types were described in the literature (cf. Ref. [6] for a summary) and strengthened the original view that  $S^3$  cells provide an independent description of dense matter properties which may capture chiral features more completely than the crystal approach, at least for  $N_c < \infty$ . (In particular, one obviously cannot view the curved  $S^3$  cells as regions of flat space containing nuclear matter, or even as being in one-to-one correspondence with unit cells of a crystal. Rather, the curvature-induced interactions in  $S^3$  were suggested to encode aspects of the dense environment in a self-sufficient way, similar in spirit to “analog-gravity” models which proved useful in many areas of physics [9].)

By claiming that the cell geometry is insignificant and that all cell shapes for Skyrmions should describe at least qualitatively similar physics, Ref. [2] therefore turns the logic of the hypersphere approach on its head. This led to the erroneous conclusions that the “evidence for chiral restoration ... was an artifact of the hyperspherical geometry” and even that “the special properties of the geometry ... make the (gained) intuition totally unreliable even for qualitative issues associated with chiral symmetry breaking and its possible restoration in the average sense”. In other words, Ref. [2] overlooks that the  $S^3$  geometry was adopted precisely for its unique symmetry properties which already the pioneering papers recognized as indispensable for chiral restoration [12]. Instead, Ref. [2] claims the opposite, namely that “this geometry has no significance” and “that using a hypersphere to restrict the volume of the Skyrmion acts generically like other restrictions on its volume”, and then suggests to discard the  $S^3$  results for not complying with these unfounded claims.

(In fact, the claim that the principal effect of the cell shape, i.e. of its topology and geometry, is just to restrict the cell volume does not even hold for flat unit cells, including those of Skyrmion crystals. The physics of the latter shows a remarkable sensitivity to the cell shape

encoded in the boundary conditions. Even the crucial half-Skyrmion symmetry originates from a subtle change in the cell form [10] and not from a generic volume restriction. The shape of unit cells in condensed-matter crystals often has a similarly decisive physical impact.)

Finally, Ref. [2] argues that all *uniformly* spatially-averaged chiral order parameters in large- $N_c$  QCD can simultaneously vanish only if chiral symmetry is also restored in the conventional, local sense, i.e. by a vanishing quark condensate. On the other hand, at least superficially the former (uniformly averaged restoration) seems to happen without the latter (local restoration) in  $S^3$  cells. There is no conflict, however, since spacial averaging over order parameters on  $S^3$  (on which in particular chiral restoration in  $S^3$  cells relies) differs qualitatively from the equal-weight averaging in flat space [13] which underlies the arguments of Ref. [2]. Indeed, there is no reason for spacial averaging over an  $S^3$  cell to translate into uniform spacial averaging over some flat-space re-

gion or configuration. Hence, Ref. [2]’s arguments imply no contradiction between the  $S^3$ -averaged hypersphere results and uniformly averaged large- $N_c$  QCD results.

To summarize: Ref. [2]’s criticism of the hypersphere approach to dense matter is based on invalid assumptions and therefore unwarranted. The  $S^3$  cell geometry, in particular, is uniquely dictated by chiral symmetry and thus an indispensable part of the approach. The qualitative differences between spacial averaging in flat space and in curved cells, furthermore, prevent any conflict with general arguments of Ref. [2] regarding “uniformly flat-space averaged” chiral restoration in large- $N_c$  QCD.

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- [1] L. McLerran and R. D. Pisarski, Nucl. Phys. A **796**, 83 (2007); Y. Hidaka, L. D. McLerran and R.D. Pisarski, Nucl. Phys. A **808**, 117 (2008).
  - [2] P. Adhikari, T.D. Cohen, R.R.M. Ayyagari and M.C. Strother, Phys. Rev. C **83**, 065201 (2011).
  - [3] I. Zahed and G.E. Brown, Phys. Rep. **142**, 1 (1986).
  - [4] N.S. Manton and P. Ruback, Phys. Lett. B **181**, 137 (1986); N.S. Manton, Commun. Math. Phys. **111**, 469 (1987).
  - [5] H. Forkel, A.D. Jackson, M. Rho, C. Weiss, A. Wirzba and H. Bang, Nucl. Phys. A **504**, 818 (1989).
  - [6] An extended and more self-contained discussion can be found in H. Forkel, arXiv:1204.2800 [nucl-th].
  - [7] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
  - [8] H. Forkel, Phys. Lett. B **280**, 5 (1992); Nucl. Phys. A **581**, 557 (1995).
  - [9] C. Barcelo, S. Liberati, M. Visser, Living Rev. Relativity, **14**, 3 (2011).
  - [10] N. S. Manton, Phys. Lett. B **192**, 177 (1987); E. Wuest, G. E. Brown and A. D. Jackson, Nucl. Phys. A **468**, 450 (1987); A. S. Goldhaber and N. S. Manton, Phys. Lett. B **198**, 231 (1987).
  - [11] Even in the quark-based Nambu-Jona-Lasinio model [7], when put into  $S^3$  cells, these curvature-induced interactions (instead of just the reduced volume) generate a transition to a chirally restored phase [8].
  - [12] This oversight becomes explicit in the calculation of Ref. [2]’s Sec. IV which is designed to show that a specific deformation of  $S^3$  prevents (averaged) chiral restoration. Awareness of the fact that any such deformation must break the crucial  $SO(4)$  symmetry of  $S^3$  makes this calculation unnecessary since the result follows directly from the symmetry arguments of Ref. [4].
  - [13] Analogous qualitative differences hold for the “translational” symmetry which the averaging procedure restores: “translations” on  $S^3$  are elements of  $SO(4)$  and do not commute, for example, in contrast to their flat-space counterparts.